TRINITY COLLEGE

Specialist Mathematics 1,2
Test 5 2017

Section 1 Calculator Free

Matrices

STUDENT'S NAME _____

DATE: Thursday 10 August **TIME:** 30 minutes **MARKS**: 34

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (6 marks)

Given $A=\begin{bmatrix}2 & 3\\ 1 & -4\end{bmatrix}$, $B=\begin{bmatrix}3 & 1\end{bmatrix}$, $C=\begin{bmatrix}-2\\ 7\end{bmatrix}$ and $D=\begin{bmatrix}4 & -2\\ 3 & 1\end{bmatrix}$, determine each of the following if possible. If not possible state why it cannot be done.

(a) $3A-D$ [2]

(b) CB [2]

(c) C^2 [2]

2. (10 marks)

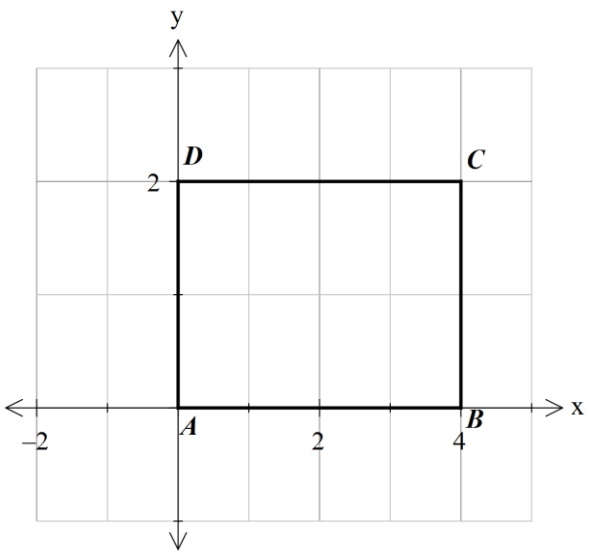
(a) Consider the matrices $A=\begin{bmatrix} 3 & 0 \\ -2 & x \end{bmatrix}$, $B=\begin{bmatrix} 2 \\ -5 \end{bmatrix}$ and $C=\begin{bmatrix} -4 & 2 \end{bmatrix}$. Determine the value of x for each of the following.

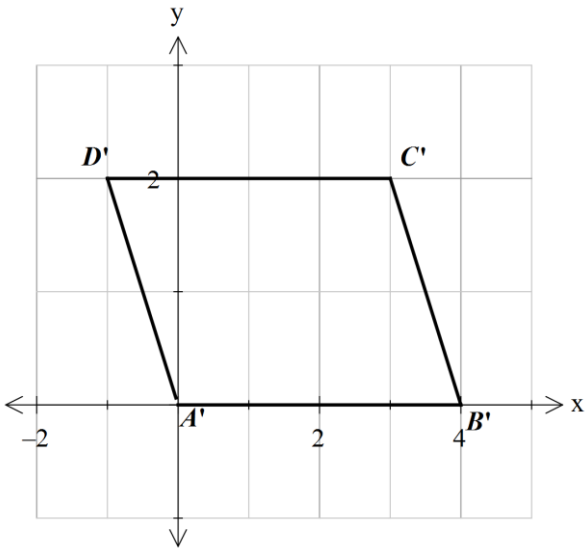
(i) $A+BC=\begin{bmatrix} -5 & 4 \\ 18 & -2 \end{bmatrix}$ [3]

(ii) $CAB=\begin{bmatrix} 12 \end{bmatrix}$ [3]

(b) If $P=\begin{bmatrix} 4 & -2 \\ 1 & 0 \end{bmatrix}$ and $Q=\begin{bmatrix} 3 & 2 \\ 6 & 4 \end{bmatrix}$ determine X given that $2XP+P=Q$ [4]

3. (5 marks)





The rectangle was transformed into a parallelogram using a shear matrix S given by

$$S = \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$$

(a) Determine the value of k [2]

(b) If the parallelogram is transformed back to a rectangle using shear matrix T, determine T. [2]

(c) What is the relationship between S and T? [1]

4. (13 marks)

(a) Let $A=\begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix}$ and $B=\begin{bmatrix} x & -2 \\ y & 5 \end{bmatrix}$. Determine the value of x and y such that A and B are commutative, i.e. $AB = BA$. [4]

(b) Given that M is a 2×2 matrix such that $M^2 = M - I$, show that $M^4 = -M$. [3]

(c) Determine the image of the line y = -2x+1 after being transformed by $\begin{bmatrix} 0 & 2 \\ -1 & 1 \end{bmatrix}$. [3]

(d) Solve for X given $X\begin{bmatrix} 4 & 0 & 4 \\ 0 & -1 & 1 \end{bmatrix} = 2\begin{bmatrix} 5 & 5 & 0 \end{bmatrix}$ [3]

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Specialist Mathematics 1,2
Test 5 2017

Section 2 Calculator Assumed
Matrices

STUDENT'S NAME _____

DATE: Thursday 10 August **TIME:** 25 minutes **MARKS**: 25

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser
Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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5. (5 marks)

If $A = \begin{bmatrix} 2 & 0 & 1 \\ -3 & 1 & 2 \\ 8 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & -1 \\ 19 & -6 & -7 \\ -8 & 0 & 2 \end{bmatrix}$

(a) calculate AB [2]

(b) solve the following set of simultaneous equations using matrix methods and the result from (a). [3]

$$x - z = -1$$
$$19x - 6y - 7z = -7$$
$$-8x + 2z = 8$$

6. (10 marks)

(a) $O(0,0)$, $A(6,1)$ and $B(5,3)$ are the vertices of a triangle. Triangle OAB is transformed to triangle $OA'B'$ where A' is (6,19) and B' is (5,18), by transformation T_1. Determine and describe matrix T_1. [3]

(b) Triangle $OA'B'$ is transformed by matrix $T_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ to triangle $OA''B''$. Determine points A'' and B''. [2]

(c) Determine a single matrix that will transform triangle OAB directly to triangle $OA''B''$. [2]

(d) Triangle $OA''B''$ is now transformed by matrix T_3 to triangle $OA'''B'''$ so that it is six times the area of the original triangle OAB. Determine three possible matrices for T_3. [3]

7. (10 marks)

Matrix $A = \begin{bmatrix} 6 & 2 \\ 3 & 7 \end{bmatrix}$

(a) Show $A^2 - 13A + 36I = 0$ where I is the identity matrix and 0 is the zero matrix. [3]

(b) Use (a) to show $A = (A - 6I)^2$. [2]

(c) Use the result from (b) to determine a square root of A. [3]

(d) Determine a second square root of *A*. [2]